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Ubiquity of the Relative Equilibrium Line Dynamic and Amplified Cyclicity in Earth Sedimentary Systems

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Abstract A distinctive feature of Earth's sedimentary systems is that they all involve the interaction between a nearly-horizontal "equilibrium line," controlling mass supply, and a dynamic sedimentary surface. For glacial systems, this is the snow line or firn line, approximating a zero-degree atmospheric isotherm. For sedimentary basin systems it is sea level or baselevel. For deep ocean carbonate sediments it is the calcite compensation depth or lysocline. First-order considerations in each case suggest a positive feedback on mass supply as the surface builds upwards (and negative feedback if the surface drops). In the first two cases, outstanding paleo-climate problems exist wherein recorded past sedimentary cycles have asymmetric amplitudes that appear too large compared to deduced vertical movements of the respective equilibrium lines. These problems are familiarly known as the "100-Kyr Pleistocene ice age cycle" and the "Myr high-order Cretaceous relative sea level cycles." Here, I discuss the emerging commonalities that surround these two amplified cycles, emphasizing the ubiquitous presence of a *relative* equilibrium line dynamic, and which for glacial systems has long been seen as providing a mass supply feedback that can reconcile the disparity between the forcing and the response. I suggest that, in the same way that continental ice sheets have been modeled as passive sedimentary systems that can freely oscillate with little or no snowline forcing, sedimentary basin systems may be capable of similar behavior without vertical sea level change and illustrate the concepts with a low-order model. Sedimentary indicators for relative sea level change may be displaying disproportionately large responses to small eustatic sea level changes, due to internal positive feedbacks.

1 Introduction

The goal of this paper is to present a framework of commonalities between two seemingly different and unrelated paleo-climate cycles. One is the Late Pleistocene 100-Kyr ice-volume cycle of Northern hemisphere ice sheets and the other is the approximately 1-3 Myr relative sea level cycle, recorded in continental margin coastal sediments. The latter cycle has been persistent throughout the Phanerozoic (past 500 Myrs) (Haq et al. 1987; Haq and Schutter, 2008).

While many readers of this journal no doubt are well acquainted with one or both of these cycles, it is worthwhile to emphasize for others the cornerstone role they each play in geoscience research, as this will underscore the need to investigate any potential link between them. One gauge may be citations of seminal papers. Two early publications that serve as landmark studies in each case are: (i) the 1976 *Science* paper by Hays et al (1976), "Variations in the Earth's Orbit: Pacemaker of the Ice Ages" about the Late Pleistocene ice sheet cycles and; (ii) the 1987 *Science* paper by Haq et al (1987), "Chronology of Fluctuating Sea Levels Since the Triassic" about relative sea level cycles on continental margins worldwide. Currently the "ISI" citation list for each paper stands at over 1,250 for Hays et al (1976) and over 2,500 for Haq et al (1987), which likely places them among the most highly cited papers in the geosciences.

Ones first reaction to a comparison of ice age ice sheet systems and sea level sedimentary systems is that they clearly involve different mass components (glacial mass from snowfall versus coastal sediments), are occurring on different timescales, and have different small-scale mechanisms for lateral mass transport. Nevertheless, on the large-scale, a number of fundamental similarities do exist. Indeed the overarching goal of this paper is to argue that rather than view one system as a strictly glacial and the other as sedimentary, both should be viewed as sedimentary, and that the anomalous cycles in each case may be different manifestations of a similar sedimentary dynamic.

The potential benefits of recognizing this similarity include suggesting new interdisciplinary modeling approaches in each case and helping decide which mechanisms, among many competing theories, are likely to be leading candidates for fundamental causes of the cycles.

Moreover, the framework makes specific predictions that are consistent with observations, such as: (i) significant glaciation did not exist during ancient greenhouse periods of the Phanerozoic, as has recently been proposed as an explanation for the puzzling high-order relative sea level cycles; (ii) an equivalent eustatic mechanism to cyclic glaciation for raising and lowering absolute sea level will not be found and; (iii) a local sediment supply instability - proposed here as a cause of relative sea level cycles - would imply the cycle could operate continuously and globally over geologic time, consistent with data (Haq and Schutter, 2008). There are also likely to be additional biogeochemical connections between the two cycles as they both intimately involve global continental margin submersion and exposure. Finally, important patterns partially fall into place such as the marked, enigmatic 'sawtooth' asymmetry of both cycles.

The following sections of the paper will first review key theory about the 100-Kyr Pleistocene ice age cycle and then about the Cretaceous Myr relative sea level cycles. Specifically these sections will: (i) revisit Milankovitch's ultimate concern with estimating paleo-snowline elevation changes in response to his orbital forcing calculations, and his conclusion that such elevation changes were insufficient, by themselves, to explain ice age cycles. (ii) I then review a class of relative snowline feedback ice sheet models that have arisen to reconcile the forcing and the response. (iii) It is argued that the basic outlines of the Pleistocene ice age problem demark the pro-typical challenges facing the analysis of any sedimentary system. From this perspective, quantifying mechanisms of global sea level change is the geologic analogue to Milankovitch's paleo-snowline elevation change calculations. (iv) A brief review is made of the problem of large Cretaceous high-frequency (~Myr) relative sea level cycles, as deduced from petroleum industry seismic sequence stratigraphy. (v) The paper rejects the possibility that greenhouse era glaciers are the cause for these enigmatic cycles, or that a yet-to-be-discovered equivalent eustatic mechanism to glaciation exists. (vi) In the absence of glacio-eustatic sea level forcing it is argued that internally driven sedimentary oscillations, analogous to the internal snowline ice sheet oscillations, can offer a solution to the disparity between sea level forcing and sedimentary response. (vii) A simple, mass-conserving, low-order model is developed to illustrate the key points and potential effects.

2 Late Pleistocene 100-Kyr Ice Sheet Cycles:

Milankovitch's Primary Concern With Snowline Elevation Changes

While the Milankovitch (1941) calculations of high-latitude insolation changes due to orbital precession, obliquity and eccentricity changes are among the most well-known pieces of work in climate science, his ultimate concern with the implications of the theory for deducing past elevations of the snowline is no doubt less widely appreciated.

The preface to “Canon of Insolation and the Ice-Age Problem” (Milankovitch, 1941), makes clear the importance of this application of the insolation changes (italics added for emphasis):

> “ …It was therefore desirable to complete my calculations of the secular march of insolation by ... *analyzing mathematically the connection between the altitude of the snowline and the radiant energy corresponding to the caloric summer half-year*. I found that a shift of the snowline by one meter corresponded to a change of this energy by one canonic unit … *With this result the most important climatic effect of the pre-historic course of terrestrial insolation, i.e. the displacement of the snow line caused by it could be determined …*”

The one-meter conversion factor was determined by a linear regression between then-available latitudinal data on summer half-year snowline elevations (data credited to Köppen as published in Wegener (1929)) and the corresponding summer half-year insolation (expressed in ‘canonic units’[1]).

Applying this conversion factor of one meter per canonic unit to the 65°N summer insolation changes implied vertical changes in the snowline there on the order of ±0.5 kilometers. Milankovitch concluded:

> "… The tables … show that the displacements of the snowline, caused directly by the change of this insolation, were powerful enough to leave clear traces of the march of insolation on the Earth's face, *but not sufficient to cause the great glaciations of prehistoric times to their full extent … For this a further climatic factor was necessary …*"

Milankovitch went on to consider ice albedo changes as the missing link. Nevertheless, it is clear that one ultimate result of Milankovitch's work was to estimate past vertical changes in a sedimentary equilibrium line -- the snowline controlling glacial mass -- and to discover that the deduced vertical movements of ~1 km appear too small by themselves to explain the full ice sheet cycles, which had lateral ice sheet extent changes measured in 1000's of kilometers.

Since the discovery of the correlation between the deep-sea oxygen isotope proxy for paleo ice-sheet *volume* and Milankovitch frequency bands (Hays et al, 1976), the modern view of the 100-Kyr cycle has in many cases shifted away from a focus on the snowline dynamic. Instead an 'input-output systems' view of the problem often dominates, where the focus is usually on the weak power in the 100-Kyr eccentricity cycle (~a few W/m^2) and the clear paradox that this cannot cause large ice sheet cycles, even though the timescales roughly match (e.g. Imbrie et al, 1993). A common summary of the puzzle is that the 100-Kyr cycle is a 'nonlinear response to Milankovitch forcing.' While this is no doubt true, the fact remains that the 100-Kyr ice sheets were created by the interaction between a snowline with an evolving glacial surface, so this dynamic must, at some level, be at the heart of the problem. It cannot be ignored.

Figure 1 is a recent example of the time-series for the 100-Kyr ice volume cycle, as indicated from the averaging of 57 worldwide benthic $\delta^{18}O$ deep sea sediment cores (Lisiecki and Raymo, 2005). The grey bars indicate the periods of rapid deglaciation, following the slower build-up of ice volume over most of the cycle, together leading to 'sawtooth' asymmetric ice volume cycles.

The same asymmetry is also seen in paleo-CO_2 and temperature records (IPCC (2007) fig 6.3). The sawtooth structure of the 100-Kyr cycle, although enigmatic, has become one of the icons of modern understanding of ice-ages. While its significance remains unexplained, its existence is favorable to the synthesis made in this paper, which sees analogies with asymmetric sedimentary cycles.

2.1 Geometric Feedback Between the Relative Snowline Position and Ice Sheet Elevation

The earliest work on the geometric feedback between the relative snowline and ice sheet elevation appears to have been made by Bodvardsson (1951). He analyzed this in connection with smaller extant glaciers on Iceland that have surface slopes on the order of 10^{-2} and did not consider it for the Laurentide ice sheets, which had surface slopes more on the order of 10^{-3}.

Still, the basic observation was that, for a nearly horizontal snowline intersecting with a large ice sheet, small vertical displacements of either the snowline, or the ice sheet surface, would result in much amplified horizontal displacements of the accumulation and ablation zones and thus significant change in the net mass supply to the sedimentary system. Furthermore the feedback was positive so that a small drop in the surface elevation (or rise in the snowline elevation) would result in an amplified expansion of the melting zone and contraction of the accumulation zone and thus decrease net mass supply. The decreased net mass supply by itself would lead to further drop in the surface elevation and so on.

The first application of this idea to the Pleistocene 100 Kyr-problem was made by Weertman (1961, 1976), who showed that it could reconcile the disparity between the ~0.5 – 1 km vertical movements in the Milankovitch insolation forcings of the snowline, with the full ice volume response. An early example of the model geometry is shown in figure 2. In this example, we illustrate the positive feedback from a small drop in surface elevation (profile 1 to profile 2), but the exact same argument would apply to a small increase in snowline elevation. As one can visualize from the horizontal and vertical length-scales, a small vertical drop in surface elevation of say ~0.5 kilometers would result in a contraction of the accumulation zone on the order of ~500 km, and expansion of the ablation zone of ~500 km, setting in motion the positive feedback described above. In later simulations, the snowline was more realistically modeled as a zero-degree atmospheric isotherm, with slow increases in elevation with decreasing latitude, but still on the order of 10^{-3}.

The basic model and feedback led to a virtual cottage industry of models during the 1980's that all employed a similar geometric framework (Källén et al, 1979; Oerlemans, 1980; Ghil and LeTreut, 1981; Pollard, 1982; Birchfield and Grumbine, 1985; Hyde and Peltier; 1985; Gaffin and Maasch, 1991). Very close simulations of the oxygen isotope proxy for ice volume (similar to the recent data shown in figure 1), including the asymmetric cycles have been obtained (e.g. Pollard, 1982). In addition, unforced cyclic behavior such as "free oscillations" have been shown – that is, the ice sheet can oscillate without any external forcing of the snowline elevation whatsoever (Källén et al, 1979; Ghil and LeTreut, 1981; Birchfield and Grumbine, 1985; Gaffin and Maasch, 1991).

Since figure 2 illustrates, by any definition, a 'passive' sedimentary system, it behooves me to ask why could not other passive sedimentary systems with equilibrium lines (e.g. passive continental margin sediments) also be able to freely oscillate if a positive feedback on mass supply exists ? Moreover, if there is a similar disparity between the deduced vertical forcing of the equilibrium line and the recorded sedimentary response, could not an analogous mass supply feedback exist to resolve the disparity ? A first attempt to illustrate the mass conservation principles involved in such an unforced sedimentary cycle was offered in Gaffin (1992).

2.2 Observing Only the *Relative* Equilibrium Line Position

The intersection of any sedimentary equilibrium line with the sedimentary surface can be referred to as the "relative" equilibrium line position, such as the relative snowline position shown in figure 2. It is easiest to think of it as the x-coordinate of the intersection point, while the y-coordinate of the intersection point can be thought of as the "absolute" equilibrium position, such as absolute snowline or absolute sea level elevation.

We can turn around the above discussion to ask what can be deduced if *only* the *relative* snowline position on an ice surface is observed, without any fixed reference frame? In other words, the reader is asked to conduct the 'thought experiment' of sitting on a glacier and watching the relative snowline position, as given by the boundary between the ablation and accumulation zones, and having no other fixed reference frame, such as a distant horizon.

It is self-evident that changes in the relative snowline position cannot be uniquely attributed to either snowline elevation or glacial surface elevation changes. Only if one can independently determine these elevation changes can the attribution be made. In this sense, I view the Milankovitch insolation changes, and snowline physics applied to them, as having independently deduced, theoretically at least, the scale for vertical movements of paleo-snowline elevations governing the Pleistocene ice sheets.

The same attribution dilemma must apply to other sedimentary systems – a geologic record of erosion and deposition is no different than the hypothetical observer sitting on a glacier, without a reference frame. Nor could any analysis of a core sample be expected to be able to solve this attribution dilemma --- consider the 'challenges' of estimating sea level changes during erosional cycles in a core, when the record itself is missing. Or the limiting case of a freely oscillating sedimentary system with no equilibrium line changes, as will be illustrated in this paper.

It is argued then that the basic outlines of the Pleistocene ice age problem demark the prototype challenges facing the analysis of any sedimentary system: one must deduce the vertical forcing of the equilibrium line controlling mass supply and compare these to the sedimentary record of the response to that forcing.

3 The Mechanisms of Global Sea Level Change on Geologic Timescales

Determining and quantifying the mechanisms of global (eustatic) sea level change for continental margins is the sedimentary analogue to Milankovitch's quantification of paleo-snowline elevation changes for ice age ice sheets. Although not adorned perhaps by the same mathematical and physical rigor of astronomical orbital calculations, such work is nevertheless playing the same role for understanding sedimentary basin cycles.

Numerous reviews have summarized the mechanisms that can raise and lower global sea level on different timescales (Miller et al, 2005). These can be categorized as those that alter either the volume of the ocean basins or the volume of water in those basins. With respect to the volume of water in the basins, the most powerful and rapid mechanism known is that shown in figure 2 – cyclic ice age ice sheet volumes, or "glacio-eustasy". The last deglaciation, shown in figure 1, raised sea level by ~120 meter in less than 20 Kyr. No other geologic mechanism is known that can affect sea level so rapidly and with such magnitudes. Therefore, if the Bodvarssen feedback is a primary cause of large amplitude glacial cycles, it is, by extension, the feedback responsible for the most rapid sea level change mechanism known.

The fact that there is no other mechanism known that can alter sea level as rapidly on the vertical scale as glacio-eustasy (e.g. Figure 1 in Miller et al (2005)), opens up the following possibility: for warm 'greenhouse' geologic periods during which there is no evidence for glaciation, which is the case for many periods of the Phanerozoic (last 500 Myrs), *we can conclude that such rapid vertical changes in sea level did not occur*. This would be the geologic analogue to the discovery by Milankovitch that past elevation changes of the snowline, due to orbital insolation changes, were insufficiently large to directly cause the ice ages, without additional feedbacks.

3.1 Cretaceous Myr Sedimentary Cycles On Passive Continental Margins

Relative sea level is the intersection of sea level with continental margin sedimentary surfaces, and will be viewed as a horizontal "x" coordinate, analogous to the relative snowline position on ice sheets, shown in figure 2. This definition of relative sea level is completely consistent with the common notion of relative sea level for coastal geomorphologists who refer to it as the combined effects of (eustatic) sea level change and subsidence or uplift of the coastal margin. The x-coordinate of the sea level position will be affected by *either* eustatic changes or coastal margin elevation changes (due to subsidence, uplift or sediment supply changes), while the y-coordinate of the sea level position will only be affected by eustatic, global sea level changes.

In the parlance of sedimentary stratigraphy, the landward and basinward movements of relative sea level are referred to as transgressions and regressions. These lateral shifts produce erosional and depositional surfaces, sequences, that can be mapped in great detail using seismic profiling, or "seismic sequence stratigraphy."

Sequence stratigraphy is a highly descriptive science, with an extensive terminology creating a disciplinary barrier. Nevertheless the physical system being studied by sequence stratigraphy is the interaction between an equilibrium line and an evolving free sedimentary surface, subject to lateral mass transport and subsidence (Gaffin and Maasch, 1991; Gaffin, 1992), the same class of system faced by Pleistocene ice sheet modelers.

The existence of large-amplitude Myr *relative* sea level cycles in passive margin sediments, persistent throughout the Phanerozoic, was first heralded in the 1970's and

1980's publications of petroleum industry analysts (Haq et al, 1987). Figure 3 shows an early example of the relative sea level curves during the Cretaceous (65-130 Myr BP). Intriguingly evident is the marked asymmetry in the transgressive and regressive cycles. That is, during the cycle, relative sea level moves landward at relatively slower rate and then abruptly basinward at the end of the cycle. The sawtooth structure of the 100-Kyr ice age cycle was noted above and here again a sawtooth structure is seen, albeit in a different sedimentary system.

The early industry synthesis reports, based on proprietary data, received critical reaction from the academic community, primarily for the attempts to equate relative sea level changes shown in figure 3 to absolute (vertical) sea level changes. This focus of the criticisms however has possibly obscured the great value of the primary data themselves – large amplitude *relative* sea level cycles.

Moreover, academic analyses, based on onshore and offshore drilling, confirms the existence of many of the same erosional and depositional cycles (Miller et al, 2005; Sahagian and Jones, 1993). The most recent of these studies have further concluded that, although controversial, they must have been caused by glacio-eustasy, given the large amplitudes of the cycles and that they appear to be synchronous on widely separated continental margins.[2] Because direct field evidence for large landed glaciers during warm greenhouse periods of the Phanerozoic, such as the mid-Cretaceous, is lacking, the proposed solution is glaciers on Antarctica that did not reach the coast. In other words, the evidence for these putative glaciers is currently hidden beneath the Antarctic ice sheet. Such analyses also concluded that either these putative glaciers existed (during much of the Phanerozoic is also implied) or there is a fundamental flaw in sea level theory (Miller et al, 2003).

A recent test for the existence of such putative glaciers, during one of the warmest intervals of the mid-Cretaceous (mid-Cenomanian ~90 Ma) has been made using the best-preserved planktic and benthic forams in a high-resolution deep-sea core (Moriya et al, 2007). The oxygen isotope records from this core show no evidence of the presumed glaciation causing the observed relative sea level cycles from sequence stratigraphy during this time. Using similar methods Bornemann et al (2008) suggest the existence of one isolated and short-lived (200,000 yr) glacial event during the mid-Cretaceous Turonian stage (~91 MyBP). Even if this is confirmed, the event itself is unconvincing as the mechanism for the multi-million year relative sea level cycles shown in figure 3, which suggest an average duration of around 1.75 million years. A recent analysis of mid-Cretaceous sea surface temperatures, using a proxy that is independent of ice volume, unlike the oxygen isotope records, suggests that oxygen isotope excursions previously attributed to sea level changes may be due to temperature changes instead (Forster et al, 2007).

3.2 What Is The Possible Alternative If Greenhouse-Era Glaciers Did Not Exist on Antarctica ?

A goal of this essay is to spur discussion of the highly plausible reality that significant glaciers did not exist on Antarctica during super-greenhouse eras, such as the mid-Cretaceous. If so, and that there is no other comparable mechanism of rapid eustatic sea level changes, what is the possible alternative explanation for large amplitude relative sea level cycles, such as shown in figure 3?

I advocate that the alternative is no more unfamiliar than the solution found by climatologists for ice age cycles – internally-driven, possibly unforced, sedimentary oscillations due to the interaction between the equilibrium line and the sedimentary surface. Consistent with this idea, a growing body of studies have shown that complex nonlinear dynamics in geomorphic and sedimentary systems may lead to dynamical instabilities which in turn lead to disproportionately large responses to changes or perturbations. A recent symposium proceedings on such complexity in geomorphology provides some references (Murray and Fonstad, 2007).

3.3 Unforced Oscillations in a Low-Order Sedimentary Basin Model

In this section we present a simple model that displays unforced oscillations resulting from a positive local feedback on sediment supply rate that is plausible for marine basins. The goal is not to try and capture all the undeniably complex aspects of coastal systems, but rather to illustrate how cycles can occur without any eustatic sea level changes. In alluvial, deltaic, and estuarine environments the link may be quite different (Phillips, 2006).[3]

Positive feedbacks between mass supply and relative sea level change are harder to see than simple area changes in snow accumulation and ablation zones on glaciers. But first-order considerations suggest they should be there. Sea level is a retarding agent in lateral sedimentary transport such that a regression basinward should lead to a local increase in mass supply rate to the marine sedimentary basin. Another way of saying this is that if relative sea level regresses, sediment that would have been deposited landward is instead available to be transported to the basin (e.g. Törnqvist et al, 2006; Phillips, 2003), thus implying a local increased mass supply rate to the basin. If this increased mass supply rate is greater than the space created beneath the depositional surface, per unit time by subsidence, the sedimentary surface must continue to build up and lead to further regression (Sloss, 1962; Gaffin 1992). This feedback could be strong enough to throw the system into oscillations without any external forcing of absolute sea level or long-range sediment supply rate changes.

I illustrate this with a mass-conserving low-order model shown in figure 4a. Lateral mass transport of terrigenous sediment into the basin is assumed proportional to surface slope (∂y/∂x) at the basin margin and a transport coefficient that depends on relative sea level position denoted κ(RSL). The central assumption is that this transport coefficient, κ**,** is weaker below sea level than it is above sea level. Figure 4b illustrates this dependence of the transport coefficient on relative sea level position (RSL). A specific functional form that gives such a shape is:

$$\kappa(\mathrm{RSL}) = \frac{1 + \mathrm{Tanh}\left[\frac{\mathrm{RSL}}{\mathrm{a}}\right]}{2} \qquad (1)$$

The parameter "a" controls the abruptness of the change in transport efficiency: small values correspond to more step-like shapes, and large values correspond to more smooth ramp-like shapes. The increasing slope of the sedimentary surface basinward, as shown in figure 4a, is consistent with this retarded transport coefficient – at equilibrium the surface slope has to increase basinward to compensate for the reduced transport efficiency.

Three time-variable model dimensions (y_1, y_2 and x_3) are measured relative to the stable, non-subsiding margin of the basin (figure 4a). y_1 is an upstream surface elevation, y_2 is the elevation of sedimentary surface at the basin edge and x_3 is the lateral extent of deposition of sediment supplied landward. Sequence stratigraphy has identified these depositional fronts and refers to them as “downlapping surfaces” (figure 3) with “condensed sections” representing sediment starvation beyond (Liu et al, 1998; Haq et al, 1987). The three degrees of freedom in this model give it sufficient geometric flexibility to simulate transgressions, regressions and downlap surfaces. Our model geometry is comparable to other low-order shapes used to study internal variability for stratigraphic sequences, including those simulated by sediment software packages (e.g. SEDPAK, Liu et al, 1998; Kim and Muto, 2006).

The mass-conserving rate equations for y_1, y_2 and x_3 are given in the Appendix. Briefly, these simply derive from the requirement that the time-rate of change of the landward volume must equal the difference between the upstream sediment flux it is receiving, “u” and the sediment flux it is losing to the basin “$-\kappa(\mathrm{RSL}).\partial y/\partial x_0$.” Similarly the time-rate of change of the basinward volume must equal the difference between the sediment it is receiving, “$-\kappa(\mathrm{RSL}).\partial y/\partial x_0$” and and the sediment it is “losing” to subsidence and sediment burial, “x_3.subsidence_rate.” To close the system dynamically, a third rate equation is needed for x_3, the depositional front (Gaffin, 1992). We make the simplest assumption that this front tends to form a fixed distance δ from RSL, with an adjustment time constant “c” (Appendix equation 6).

In these simulations, the upstream supply rate of sediment (“u”) and absolute sea level elevation (SL) are held strictly constant. An equilibrium state would obtain if, per unit time, the same volume of sediment is being buried within the basin, as is being supplied upstream (u). The question studied here is, what is the stability of this equilibrium state if the transport coefficient depends on RSL as shown in figure 4b?

We start the model in equilibrium by setting the upstream flux equal the initial value of x_3 times the basin subsidence rate. Equations (4), (5) and (6) in the Appendix are the mass conserving rate and transport equations that are integrated. The two main adjustable parameters for the model are “a” which controls the transport non-linearity and “c” which controls the adjustment time for the depositional front.

Upon integration we quickly find unforced, self-sustained oscillations. An example is shown in figure 5. This figure shows the position of relative sea level (RSL) due to oscillations in the three model dimensions y_1, y_2 and x_3. The unforced oscillations are due to an internal instability introduced by the nonlinear transport coefficient -- if RSL moves landward, the margin becomes submerged and less vigorous lateral transport takes place locally. This decreases mass supply rate to the basin volume which causes dimension y_2 to drop. The decrease in y_2 then leads to further transgression. Since the upstream flux of sediment, u, is constant, elevation y_1 is forced to increase due to the reduction in mass loss to the basin. Dimension x_3 also moves landward to track RSL. At some point the increase in y_1 and decrease in x_3 forces y_2 to increase and RSL to regress, setting in motion the same instability but in the opposite direction.

The space and time units are unscaled in these simulations, but can easily be assigned units to match the spatial and time scales observed in actual sedimentary cycles. For example, in figure 5 if we assign a value of 1000 years to the time step and 100 kilometers to the horizontal spatial unit, the simulated cycles then have a duration of ~1

Myrs, with relative sea level transgressing and regressing distances of a few hundreds of kilomters – this is in general agreement with published data on the cycles (e.g. Vail et al, 1977).

The simulated cycles also have a sawtooth asymmetry as indicated by the actual data (figure 3). While this may be a tentative agreement, it does show that auto-oscillating systems can easily produce asymmetric cycles and any full explanation of the coastal onlap cycles will have to confront the asymmetry since it is such a distinctive feature of the record (figures 3). Although the model is low-order, we would expect a full two-dimensional partial-differential equation model, using a similar non-linear transport coefficient, would show analogous cycles.

4 Conclusions

This paper has argued that, as the problems of high-frequency relative sea level cycles during warm geologic epochs become more acute, it will be inevitable to make a comparison between the problems of amplified cyclicity in glacial sedimentary systems and in passive margin sedimentary systems. Although the materials and transport processes in each case are different, the large-scale dynamics involved are quite similar -- the geometric interaction between an evolving free sedimentary surface (subject to retarded lateral mass transport and subsidence) and a nearly horizontal equilibrium line. This geometry implies that small vertical changes in the equilibrium line will be amplified into much greater horizontal changes in the relative equilibrium line, with attendant effects on mass supply rates. Moreover the geologic record in each case is showing asymmetric cycles that appear too large compared to the small deduced vertical forcings of the equilibrium line. The ‘small deduced vertical forcing’ of the snowline was the ultimate result from Milankovitch for his orbital calculations. The ‘small deduced vertical forcing’ of sea level is the *default* result from the absence of a known mechanism of sea level change comparable to glaciation during warm geologic epochs, such as the Cretaceous, when large ice sheets were very unlikely to exist. The relative equilibrium line feedback on mass supply appears to be positive in both cases. This positive feedback has been the basis for a large class of ice sheet models that have reconciled weak Milankovitch forcing with the ice sheet response on 100-Kyrs, including unforced oscillations. There seems to be no reason why a similar positive feedback on mass supply cannot lead to unforced or weakly forced oscillations of large amplitude in passive margin sedimentary systems too. A local instability of this nature could also imply that the cycle should be continuously and globally operative over geologic time.

One way or another, the two paleo-climate cycles discussed will be linked somehow. Either they are both glacio-eustatic, as is the default conclusion of some analysts (Miller et al, 2005) or, if no Cretaceous glaciers existed, they are both sedimentary cycles – the latter case presented here.

Appendix

Dimensions y_1, y_2 and x_3 are time-variable and governed by mass conservation. The upstream flux of sediment, u, the absolute sea level elevation, SL and the basin subsidence rate are all held constant.

Transport of sediment into the basin is assumed proportional to the sedimentary surface slope at the margin, $\partial y/\partial x_0$, and a transport coefficient κ(RSL) which depends on relative sea level, RSL. The central assumption in the model is that κ(RSL) has a generic shape as illustrated in figure 4b. This shape is consistent with a gradually increasing sediment slope that is observed within continental margin sedimentary systems – at equilibrium the surface slope must increase basinward to overcome the retarded transport coefficient. The specific functional form chosen for κ(RSL) is:

$$\kappa(\mathrm{RSL}) = \frac{1 + \mathrm{Tanh}\left[\frac{\mathrm{RSL}}{\mathrm{a}}\right]}{2} \qquad \text{(A1)}$$

The parameter "a" controls the abruptness of the nonlinear transport efficiency.

RSL at any time is simply the intersection point of the surface with the fixed SL elevation. $\partial y/\partial x_0$, RSL and κ(RSL) depend at each moment on the concurrent values of y_1, y_2 and x_3 and the retarded transport coefficient assumption.

Assuming the sedimentary surfaces are approximately piecewise linear, then the volume of sediment landward of the margin is given by:

$$Volume_{landward} = \frac{1}{2}[y_1 + y_2] \cdot x_1 \qquad \text{(A2)}$$

where x1 is held constant. The volume of sediment basinward of the margin is given by:

$$Volume_{ba\sin ward} = \frac{1}{2}[y_2 \cdot x_3] \tag{A3}$$

The time-rate of change of the landward volume must equal the difference between the (constant) upstream flux of sediment, u, and the flux of sediment entering the basin, $-\kappa(RSL).\partial y/\partial x_0$:

$$\frac{1}{2}[\dot{y}_1 + \dot{y}_2] \cdot x_1 = u + \kappa(RSL) \cdot \frac{\partial y}{\partial x_0} \tag{A4}$$

where the over-dot refers to the time-derivative and the subscript 0 refers to the slope at the basin margin. Similarly, the time-rate of change of the basinward volume must equal the difference between the sediment supplied $-\kappa(RSL).\partial y/\partial x_0$ and that being buried due to subsidence:

$$\frac{\dot{y}_2 \cdot x_3}{2} + \frac{\dot{x}_3 \cdot y_2}{2} = -\kappa(RSL) \cdot \frac{\partial y}{\partial x_0} - x_3 \cdot subsidence\_rate \tag{A5}$$

A final rate equation is needed for x_3, the changing depositional front of sediment that we interpret corresponds to the downlap surfaces described in sequence stratigraphy (Gaffin, 1991; Haq et al, 1987). We make the assumption that this front tends to form at a fixed distance, δ, from the relative sea level position, RSL. This simple dynamic can be modeled with the following adjustment-time rate equation:

$$\dot{x}_3 = \frac{1}{c}(RSL + \delta - x_3) \tag{A6}$$

Where c is an unknown adjustment time constant governing how quickly the depositional front moves to a distance δ from RSL. In other words, if RSL regresses basinward, the right-hand-side of (5) will be positive and this will cause x_3 to increase, trying to restore equilibrium with an adjustment time given by coefficient c.

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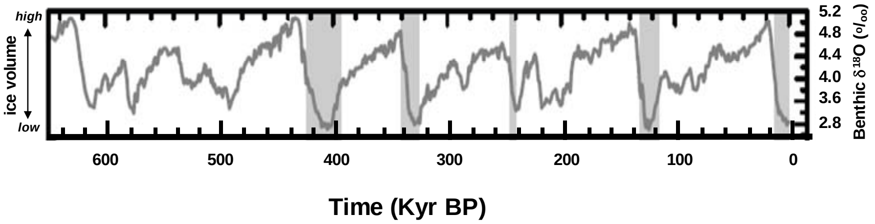

FIGURE 1

Figure 1: A recent example of the time-series for the 100-Kyr ice volume cycle, as indicated from the 'stacking' of 57 worldwide benthic δ^{18}O deep sea sediment cores (Lisiecki and Raymo, 2005; as reproduced in figure 6.3 in the IPCC Working Group 1 4th Assessment Report, Chapter 6, page 444). The grey bars indicate the periods of rapid deglaciation following the much slower build-up of ice volume over most of the cycle, leading to 'sawtooth' asymmetric ice volume cycles.

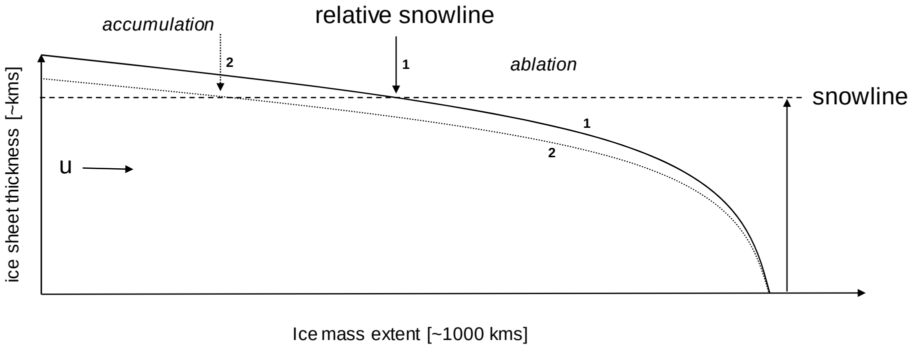

FIGURE 2

Figure 2: Cross-sectional schematic of Northern hemisphere ice age ice sheet intersecting with a snowline. Weertman (1961) first proposed this model to illustrate the internal relative snowline feedback on mass supply and instability that can exist between an ice sheet elevation and the *relative* snowline position. In profile 1, accumulation of snow is occurring northward the *relative* snowline position, while ablation is occurring southward. Assume this profile is in mass balance. If, for whatever reason, the elevation drops to profile 2, there will be a northward expansion of the ablation surface area, and a similar contraction of the accumulation surface area, as illustrated. This implies a reduction in net mass supply to the glacier, negative mass balance, and will lead to further drop in surface elevation and so on. This basic instability led to large class of ice sheet simulations that can reconcile the weak Milankovitch Northern hemisphere insolation changes with the very strong 100-Kyr ice volume cycles shown in figure 1, including asymmetric cycles.

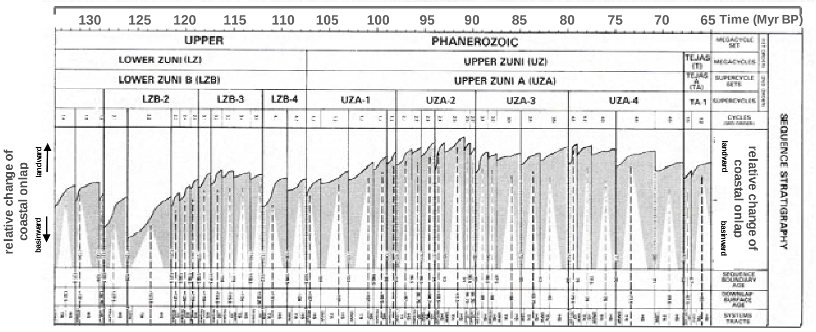

FIGURE 3

Figure 3: (From Haq et al (1987). Reprinted with permission from AAAS.) Relative sea level cycles during the Cretaceous period (65-135 Myr BP) as inferred from seismic sequence stratigraphy. Note the spatial designation of “landward” and “basinward” indicating these cycles represent the horizontal component of sea level changes. Similar cycles seem to have been persistent throughout the Phanerozoic (past 550 Myrs) (Haq and Schutter, 2008). The cycles clearly indicate an asymmetry with gradual onlapping of relative sea level landward followed by abrupt regression basinward.

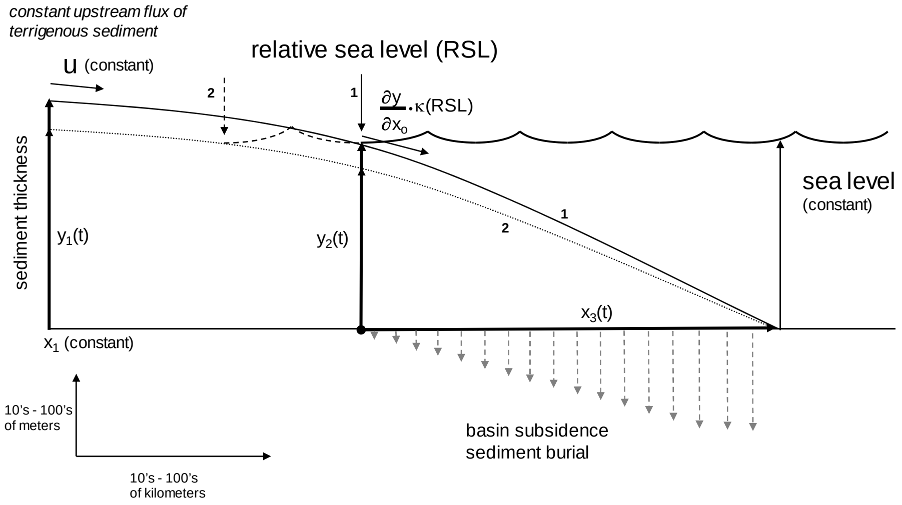

FIGURE 4a

Figure 4a: Low-order passive margin sediment model used to simulate unforced oscillations in relative sea level. "u" is an upstream source rate of terrigenous sediment that is held constant. SL is the "absolute" elevation of sea level, relative to a stable, non-subsiding continental reference frame and is also held fixed. The downward arrows illustrate basin subsidence into which the sediments are being deposited and ultimately buried. This rate is also held fixed. Sediments are assumed to flux into the basin at a rate dependent on the surface slope at the margin, $\partial y/\partial x_0$, and a transport coefficient, κ(RSL). The central assumption is that this transport coefficient, **κ,** is weaker below sea level than it is above sea level. Figure 4b illustrates this dependence of the transport coefficient on relative sea level position (RSL). Three time-variable model dimensions (y_1, y_2 and x_3) are measured relative to the stable, non-subsiding margin of the basin: y_1 is an upstream surface elevation, y_2 is the elevation of sedimentary surface at the basin edge and x_3 is the lateral extent of deposition of sediment supplied landward. The appendix derives simple mass conservation rate equations for y_1, y_2 and x_3 by considering the mass supply and loss rates for the landward and basinward volumes.

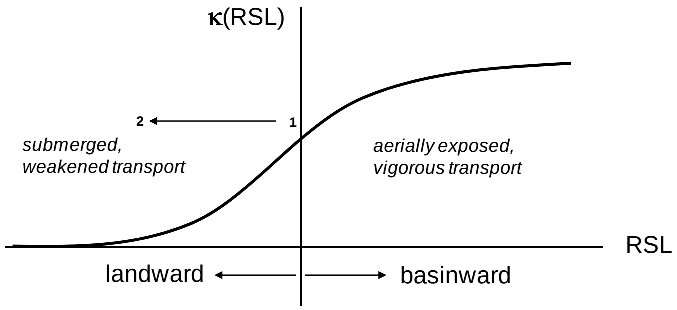

FIGURE 4b

Figure 4b: Assumed dependence of the sediment transport coefficient κ(RSL) on RSL.

The physical rationale for this dependence is that if RSL moves landward (profile 1 compared to profile 2 in figure 4a), the basin margin becomes submerged and sediments are subject to less vigorous fluvial transport and weathering agents. Alternatively, sediments that are deposited landward with a transgression, are unavailable to be transported to the basin, reducing sediment supply rates to the basin.

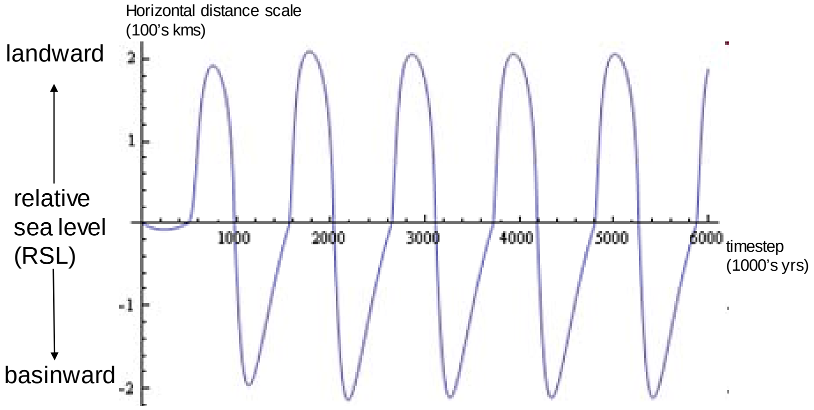

FIGURE 5

Figure 5: Unforced asymmetric oscillations in RSL from the model. These are resulting from the local sediment supply feedback introduced by the retarded transport coefficient assumed in figure 4b. The model was run in arbitrary unscaled units, but we can assign a value of ~1000 years to the time step and ~100 kilometers to spatial scale to mimic the Myr cycles shown in figure 3. A local sediment supply instability would imply the cycle could operate continuously and globally over geologic time, consistent with data (Haq and Schutter, 2008).

ENDNOTES

[1] Milankovitch's 'canonic unit' of radiation was the solar constant (1355 W/m^2) and canonic time units were 10^{-5} year.

[2] This paper holds the position that no sedimentary-core analysis would have the precision and control over all variables to untangle the attribution dilemma between

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sedimentary equilibrium line and surface elevation changes. In support of this, one can consider the ‘challenges’ of estimating sea level changes during erosional cycles in a core, when the record itself is missing. Or the limiting case, as illustrated in this paper, of a freely oscillating sedimentary system with no vertical sea level changes.

[3] In some cases fluvial inputs to coastal and marine systems have been shown to be remarkably constant over long time periods despite known changes in sediment production within the drainage basins driven by climate and other factors, either because some master factor (such as tectonic uplift) overwhelms other factors that influence sediment yield, or because alluvial storage buffers the effects of changes in sediment production on sediment yield. This has the effect of damping or obscuring glacio-eustatic or other eustatic signals (Granger et al (1996); Métivier and Gaudemar (1999); Phillips (2003); Phillips and Slattery (2006); Summerfeld and Hulton (1994)).